Name: ________________________________ Date:____________

A2 CC1 Rate of Change

1 Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of B dollars after m months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after m months.

m (x)	B (y)
0	1000.00
10	1172.00
19	1352.00
36	1770.80
60	2591.90
69	2990.00
72	3135.80
73	3186.00

50 1419.9

50 1638

13

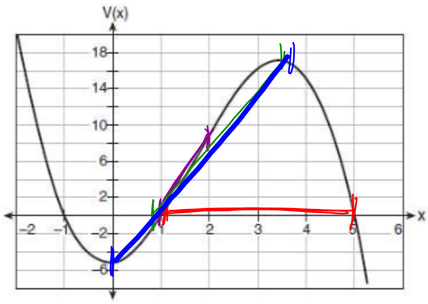
36 1365

594.1

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

1) month 10 to month 60 28.39
2) month 19 to month 69 32.76
3) month 36 to month 72 37.91
4) month 60 to month 73 45.7

2 A cardboard box manufacturing company is building boxes with length represented by $x+1$, width by $5-x$, and height by $x-1$. The volume of the box is modeled by the function below.



Over which interval is the volume of the box changing at the fastest average rate? Steepest

1) [1,2]
2) [1,3.5]
3) ~~[1,5]~~
4) [0, 3.5]

3 The value of a new car depreciates over time. Greg purchased a new car in June 2011. The value, V, of his car after t years can be modeled by the equation $\log_{0.8}\left(\frac{V}{17000}\right)=t$. What is the average decreasing rate of change per year of the value of the car from June 2012 to June 2014, to the *nearest ten dollars per year*?

1) 1960
2) 2180
3) 2450
4) 2770

4 The function $f(x) = 2^{-0.25x} \bullet \sin\left(\frac{\pi}{2}x\right)$ represents a damped sound wave function. What is the average rate of change for this function on the interval $[-7,7]$, to the *nearest hundredth*?

1) -3.66
2) -0.30
3) -0.26
4) 3.36

5 The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.

Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

Be

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

6 Which function shown below has a greater average rate of change on the interval $[-2,4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

AVERAGE RATE OF CHANGE
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. For the function $g(x)$ given in the table below, calculate the average rate of change for each of the following intervals.

x	-3	-1	4	6	9
$g(x)$	8	-2	13	12	5

(a) $-3 \le x \le -1$

$$\frac{g(-1)-g(-3)}{-1-(-3)}=\frac{-2-8}{2}=\frac{-10}{2}=-5$$

(b) $-1 \le x \le 6$

$$\frac{g(6)-g(-1)}{6-(-1)}=\frac{12-(-2)}{7}=\frac{14}{7}=2$$

(c) $-3 \le x \le 9$

$$\frac{g(9)-g(-3)}{9-(-3)}=\frac{5-8}{12}=\frac{-3}{12}=-\frac{1}{4}$$

(d) Explain how you can tell from the answers in (a) through (c) that this is **not** a table that represents a linear function.

If this was a linear function then the average rate of change would have been the same for each of these intervals. Since the average rate of change changes, this is not a linear function.

2. Consider the simple quadratic function $f(x)=x^2$. Calculate the average rate of change of this function over the following intervals:

(a) $0 \le x \le 2$

$$\frac{f(2)-f(0)}{2-0}=\frac{4-0}{2}=\frac{4}{2}=2$$

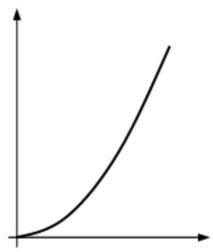
(b) $2 \le x \le 4$

$$\frac{f(4)-f(2)}{4-2}=\frac{16-4}{2}=\frac{12}{2}=6$$

(c) $4 \le x \le 6$

$$\frac{f(6)-f(4)}{6-4}=\frac{36-16}{2}=\frac{20}{2}=10$$

(d) Clearly the average rate of change is getting larger at x gets larger. How is this reflected in the graph of f shown sketched to the right?



The graph is getting steeper as we move from left to right.

3. Which has a greater average rate of change over the interval $-2 \le x \le 4$, the function $g(x) = 16x - 3$ or the function $f(x) = 2x^2$? Provide justification for your answer.

Because the function $g(x)$ is a linear function, its average rate of change over any interval will be its slope. For $g(x)$ this is equal to 16.

AROC = 16

$f(-2) = 2(-2)^2 = 2(4) = 8$
$f(4) = 2(4)^2 = 2(16) = 32$
$\frac{f(4)-f(-2)}{4-(-2)} = \frac{32-8}{6} = \frac{24}{6} = 4$

$\frac{32-8}{4-(-2)} = \frac{24}{6} = 4$

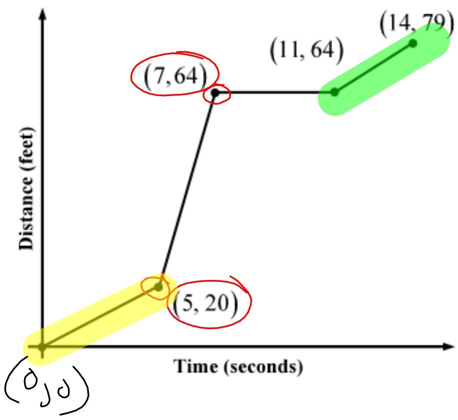
So, clearly, $g(x)$ has the greater average rate of change by far since its average rate is 16 and the average rate of $f(x)$ is only 4.

APPLICATIONS

4. An object travels such that its distance, *d*, away from its starting point is shown as a function of time, *t*, in seconds, in the graph below.

(a) What is the average rate of change of *d* over the interval $5 \le t \le 7$? Include proper units in your answer.

$\frac{64 \text{ ft} - 20 \text{ ft}}{7 \text{ sec} - 5 \text{ sec}} = \frac{44 \text{ ft}}{2 \text{ sec}} = 22 \frac{\text{ft}}{\text{sec}}$



(b) The average rate of change of distance over time (what you found in part (a)) is known as the **average speed** of an object. Is the average speed of this object greater on the interval $0 \le t \le 5$ or $11 \le t \le 14$? Justify.

For $0 \le t \le 5$:
$\frac{20-0}{5-0} = \frac{20 \text{ ft}}{5 \text{ sec}} = 4 \frac{\text{ft}}{\text{sec}}$

For $11 \le t \le 14$:
$\frac{79-64}{14-11} = \frac{15 \text{ ft}}{3 \text{ sec}} = 5 \frac{\text{ft}}{\text{sec}}$

The average speed is slightly greater on the interval $11 \le t \le 14$.

REASONING

5. What makes the average rate of change of a linear function different from that of any other function? What is the special name that we give to the average rate of change of a linear function?

The average rate of change is a constant for linear functions and does not depend on the interval over which it is calculated. No other type of function behaves this way. We call the average rate of change of a linear function its slope.